Hilroy

80 Pages

27.6 cm x 21.2 cm

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EXERCISE BOOK
CAHIER D'EXERCICES

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SUBJECT/SUJET ELEC 211 MATH 264

ASSEMBLED IN CANADA WITH IMPORTED MATERIALS
ASSEMBLÉ AU CANADA AVEC DES MATIÈRES IMPORTÉES

12107

Potential

1/15/19
LEC

Work

$F = QE$

"if we move a particle in direction $\hat{a}_L$

$F = Q\vec{E}\cdot\hat{a}_L$

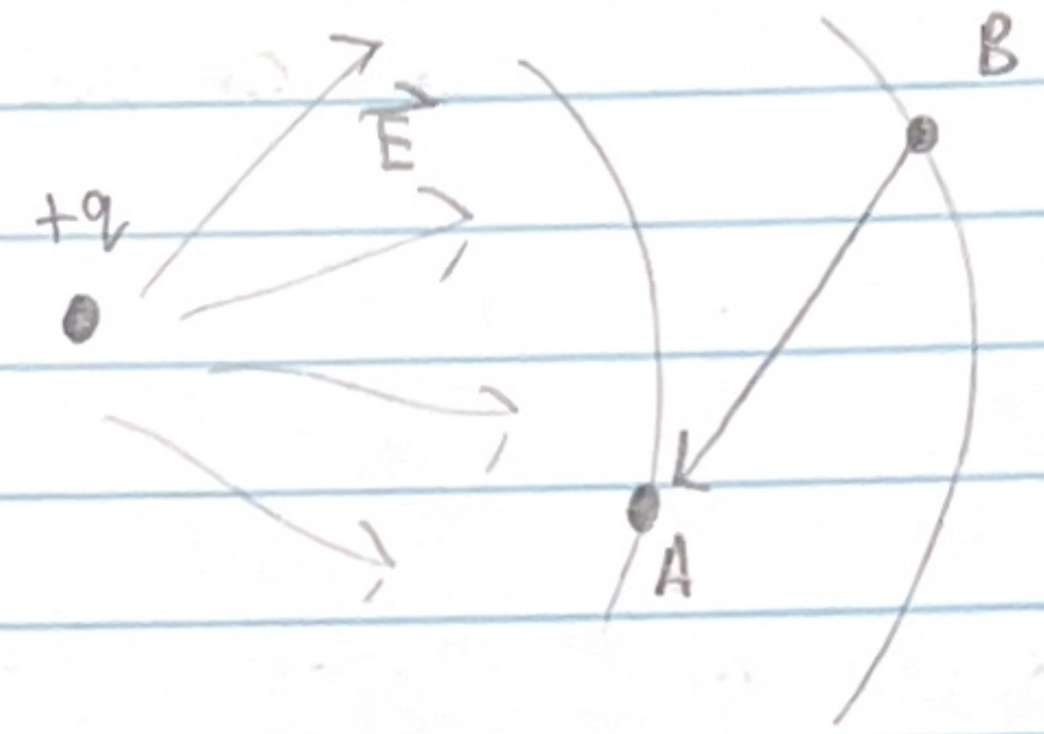
$$W = -q\int_{initial}^{Final} E\cdot dL = 0$$

Work between two charges:

$$W = \frac{qQ}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

Potential

$$\text{potential difference} = V = -\int_{initial}^{Final} E\cdot dL$$



$$V = -\int_{B}^{A} \vec{E}\cdot d\vec{L}$$

Potential => Scalar. usually a function of distance

Q	$\lvert E \rvert$	$\lvert V \rvert$
Point charge	$\frac{Q}{4\pi\epsilon_0 R^2}$ $\left(\propto \frac{1}{R^2}\right)$	$\frac{Q}{4\pi\epsilon_0 R}$ $\left(\propto \frac{1}{R}\right)$
Line charge	$\frac{\rho_L}{2\pi\epsilon_0 \rho}$ $\left(\propto \frac{1}{\rho}\right)$	$\left(\propto \frac{1}{\ln(\rho)}\right)$
Sheet of Charge	$\frac{\rho_L}{2\epsilon_0}$ (constant)	$\propto n$ Where: n is normal to the sheet

Math 264

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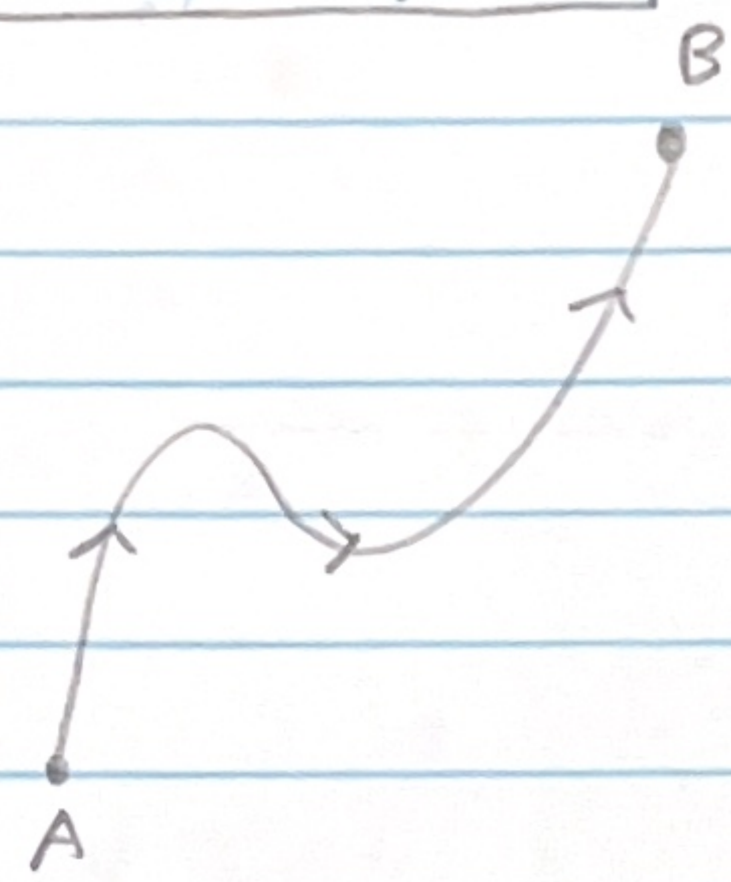
Line Integral



"let L be a curve in space with starting point A and endpoin B

Then we define a line integral over L of a <u>scalar</u> function

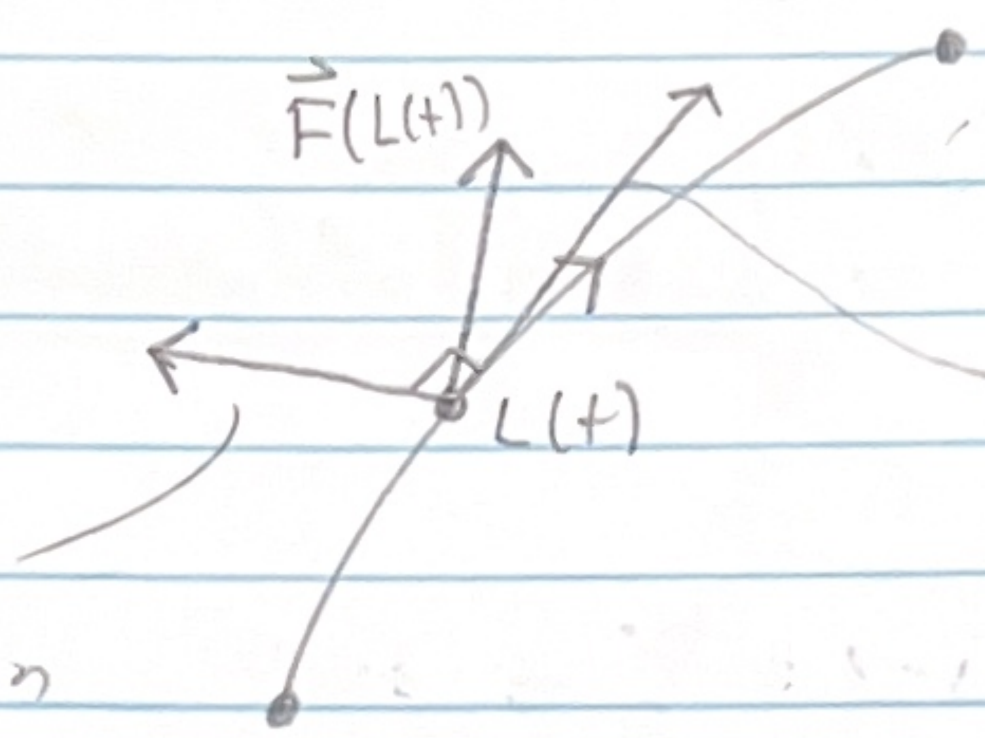
as $\int_L f\,ds$ "ex: to calculate the total charge with variable density"

or define it as a vector function $\vec{F}(x,y,z)$

$\int_L \vec{F} \cdot d\vec{b}$ "ex: to calculate work done by a force field along L

differential line form

In defining these the parameterization of the curve is very important



no affect on work

- Curve (C)
- let $L(t)$ be the parameterization of C
- direction of the curve, we can find by $\frac{dL}{dt}$ (gradient)

→ The component of $\vec{F}(L(t))$ along C call

$\vec{F}_c$ is

$$|\vec{F}_c| = \vec{F} \cdot \underbrace{\hat{\frac{dL}{dt}}}_{\text{unit vector along } \frac{dL}{dt}}$$

↳ tiny piece of work done

$$dW = |\vec{F}_c| \cdot \overbrace{ds} = \left(\vec{F} \cdot \hat{\frac{dL}{dt}}\right) \cdot \left|\frac{d\vec{L}}{dt}\right|$$

ds

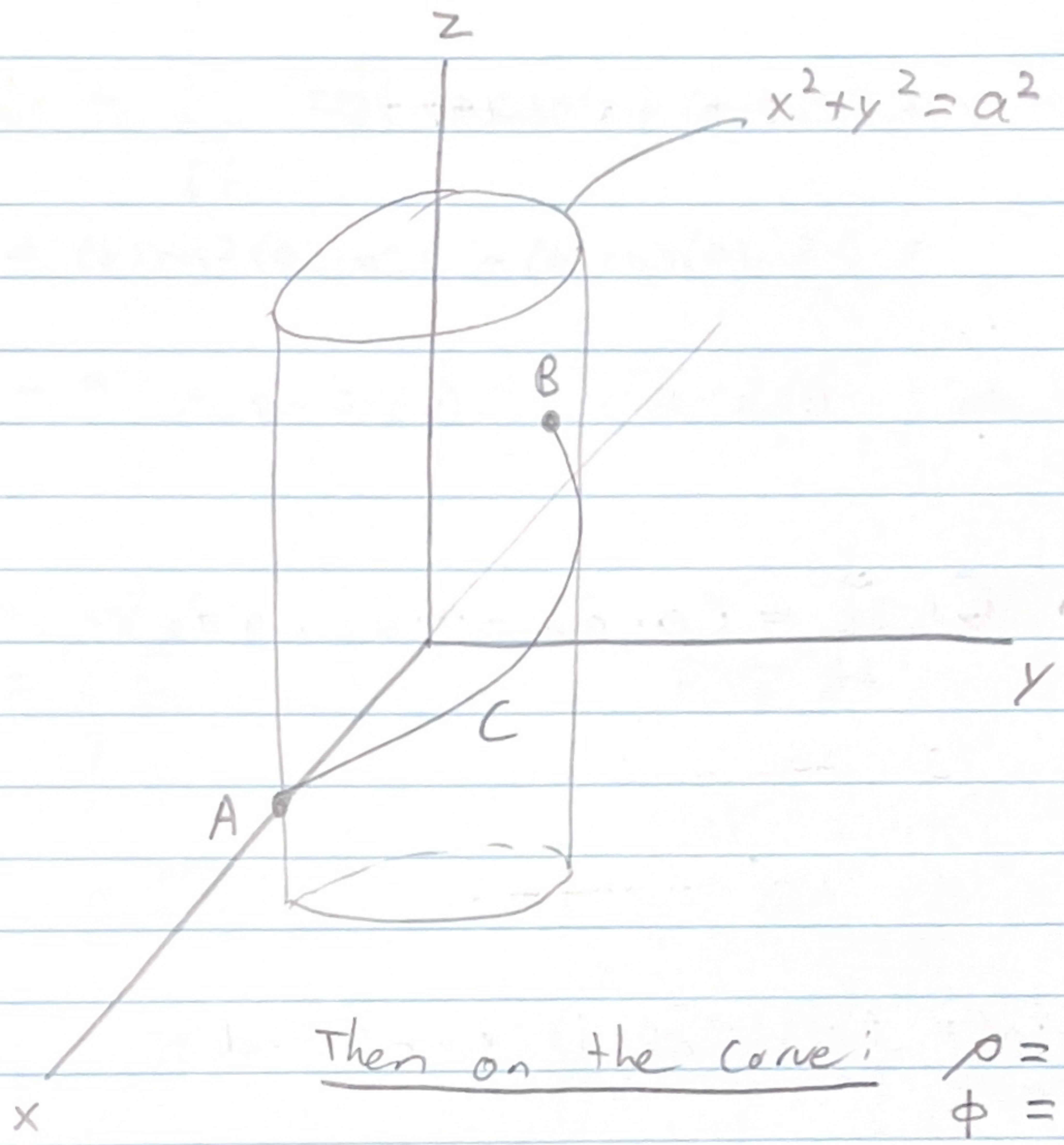
$$= \vec{F} \cdot \frac{\frac{d\vec{L}}{dt}}{\cancel{\left|\frac{dL}{dt}\right|}} \cdot \cancel{\left|\frac{d\vec{L}}{dt}\right|}$$

$$\boxed{dW = \vec{F}(L(t)) \cdot \frac{d\vec{L}}{dt}}$$

↳ $$\boxed{W = \int_C \left(\vec{F} \cdot \frac{d\vec{L}}{dt}\right) dt}$$

$\boxed{\text{In Cylindrical Coordinates}}$

Same problem using Cylindrical



Then on the curve: $\left.\begin{array}{l}\rho = a\\ \phi = t\\ \theta = kt\end{array}\right]$ $0 < t < \pi$

Then $\frac{dL}{dt} = \hat{a}_\rho \frac{d\rho}{dt} + \rho \hat{a}_\phi \frac{d\phi}{dt} + \hat{a}_z \frac{dz}{dt}$

$\hookrightarrow \frac{dL}{dt} = \hat{a}_\rho \cdot 0 + \hat{a}_\phi \cdot a \cdot 1 + \hat{a}_z \cdot k$

now write $\vec{G}$ in Cylindrical coordinates

$\vec{G} = G_\rho \hat{a}_\rho + G_\phi \hat{a}_\phi + G_z \hat{a}_z$

↳ $G_\rho = \vec{G} \cdot \hat{a}_\rho = \hat{a}_\rho \cdot (y\hat{a}_x - x\hat{a}_y - g\hat{a}_z)$

$= y(\hat{a}_\rho \cdot \hat{a}_x) - x(\hat{a}_\rho \cdot \hat{a}_y) - g\cancelto{0}{(\hat{a}_\rho \cdot \hat{a}_z)}$

$= y\cos(\phi) - x\sin(\phi) - 0$

$= a\sin(\phi)\cos(\phi) - a\cos(\phi)\sin(\phi) = 0$

Similarly: $G_\phi = -a$; $G_z = -g$

↳ $\vec{G} \cdot \frac{d\vec{L}}{dt} = \langle 0 \cdot \hat{a}_\rho - a \cdot \hat{a}_\phi - g\hat{a}_z \rangle \cdot (a\hat{a}_\phi + k\hat{a}_z)$

$= -a^2 - gk$

$W = \int_0^\pi (-a^2 - gk)\,dt = -a^2\pi - gk\pi$

$\boxed{\text{Scalar Integral}}$

Let C carry static charge with linear density $\rho_L = \alpha(x^2 + z^2)$

Find Total charge (C = HELIX)

$Q_T = \int_C dQ$ where: $dQ = \rho_L \cdot ds$

$Q_T = \int_C \rho_L\, ds$ where: $ds = \left|\frac{dL}{dt}\right|$

ELECTRIC POTENTIAL

1/22/18
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let $\vec{r} = \langle x, y, z \rangle$; $r = |\vec{r}|$ and let $g = \frac{c}{r}$
; c is a constant

$$g = c(x^2+y^2+z^2)^{-1/2}$$

$$\nabla g = \left\langle \frac{dg}{dx}, \frac{dg}{dy}, \frac{dg}{dz} \right\rangle$$

$$\frac{dg}{dx} = c \cdot -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{dg}{dy} = \frac{-cy}{(x^2+y^2+z^2)^{3/2}}$$

$$\frac{dg}{dz} = \frac{-cz}{(x^2+y^2+z^2)^{3/2}}$$

$$\vec{\nabla} g = -c \frac{\langle x, y, z \rangle}{(x^2+y^2+z^2)^{3/2}}$$

$$= -c \cdot \frac{1}{x^2+y^2+z^2}, \frac{\langle x, y, z \rangle}{(x^2+y^2+z^2)^{1/2}}$$

$$= \frac{-c}{r^2} \cdot \frac{\vec{r}}{|\vec{r}|}$$

choose $c = \frac{Q}{4\pi\epsilon_0}$

$\hookrightarrow -\vec{\nabla} g = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{r^2} \cdot \hat{a}_r = \vec{E}$ (For a point charge)

Example

We have a line of charge on z axis with uniform density ρ_L. Find the potential satisfying $-\vec{\nabla} V = \vec{E} = \frac{\rho_L}{2\pi\epsilon_0} \cdot \frac{1}{\rho} \hat{a}_\rho$

$\rho = \sqrt{x^2+y^2}$ $\quad\quad$ $\hat{a}_\rho = \frac{\langle x, y, 0\rangle}{\sqrt{x^2+y^2}}$

$$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0} \cdot \frac{1}{\sqrt{x^2+y^2}} \cdot \frac{\langle x, y, 0\rangle}{\sqrt{x^2+y^2}}$$

$$\vec{\nabla} V = \frac{-\rho_L}{2\pi\epsilon_0} \left\langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}, 0 \right\rangle$$

$$\frac{dV}{dx} = \frac{-\rho_L}{2\pi\epsilon_0} \cdot \frac{x}{x^2+y^2}$$

$$V = \frac{-\rho_L}{2\pi\epsilon_0} \int \frac{x}{x^2+y^2}\, dx$$ — SUBSTITUTION METHOD

$$V = \frac{-\rho_L}{2\pi\epsilon_0} \cdot \frac{1}{2} \ln(x^2+y^2) + \boxed{C}$$ — $g(y,z)$

$$V = \frac{-\rho_L}{2\pi\epsilon_0} \cdot \frac{1}{2} \ln(x^2+y^2) + C$$

$$\frac{dV}{dy} = \frac{-\rho_L}{2\pi\epsilon_0} \cdot \frac{1}{2} \cdot \frac{2y}{x^2+y^2} + \frac{dC}{dy}$$

By math $\frac{dC}{dy} = 0$

$\therefore g(y,z) = k(z)$ independent of y

$$\frac{dV}{dz} = 0 = \frac{dk}{dz}$$

$\therefore$ $k(z) =$ a constant

constant

$$V = \frac{-\rho_L}{2\pi\epsilon_0} \cdot \frac{1}{2} \ln(x^2+y^2) + \boxed{C}$$

$$V = \frac{-\rho_L}{2\pi\epsilon_0} \cdot \ln\left(\sqrt{x^2+y^2}\right) + C$$

$$V = \frac{-\rho_L}{2\pi\epsilon_0} \ln(\rho) + C$$

$$V = \frac{\rho_L}{2\pi\epsilon_0} \cdot \ln\left(\frac{1}{\rho}\right) + C$$

$\vec{F} = \vec{\nabla} f$. Let C be a curve from A to B. Let $L(t)$ be a parameterization of C with $L(a) = A$
$L(b) = B$

Then $$\int_C \vec{F} \cdot d\vec{L} = \int_a^b \left(\vec{F}(L(t)) \cdot \frac{dL}{dt}\right) dt$$

$$= \int_a^b \left(\vec{\nabla} f(L(t)) \cdot \frac{dL}{dt}\right) dt$$

$$= \int_a^b \frac{d}{dt} f(L(t))$$

$$= f(L(b)) - f(L(a))$$

Gauss Law

1/24/19
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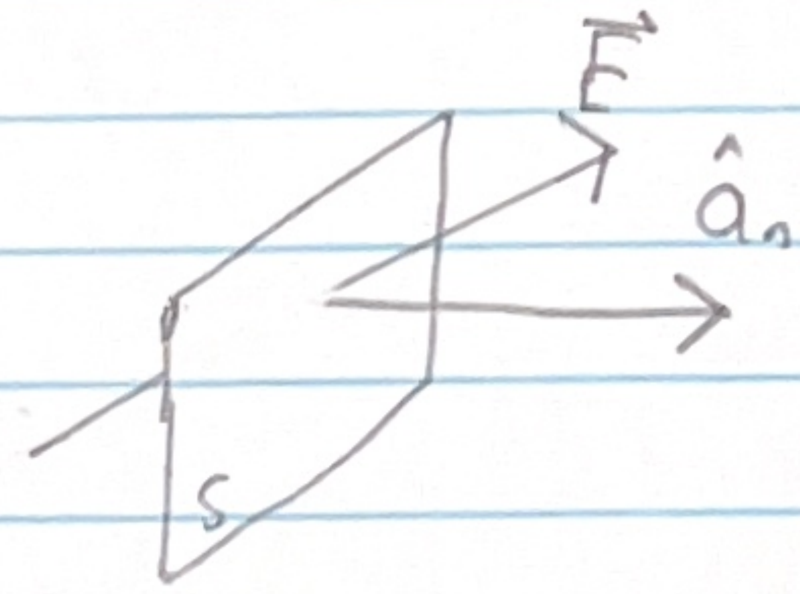
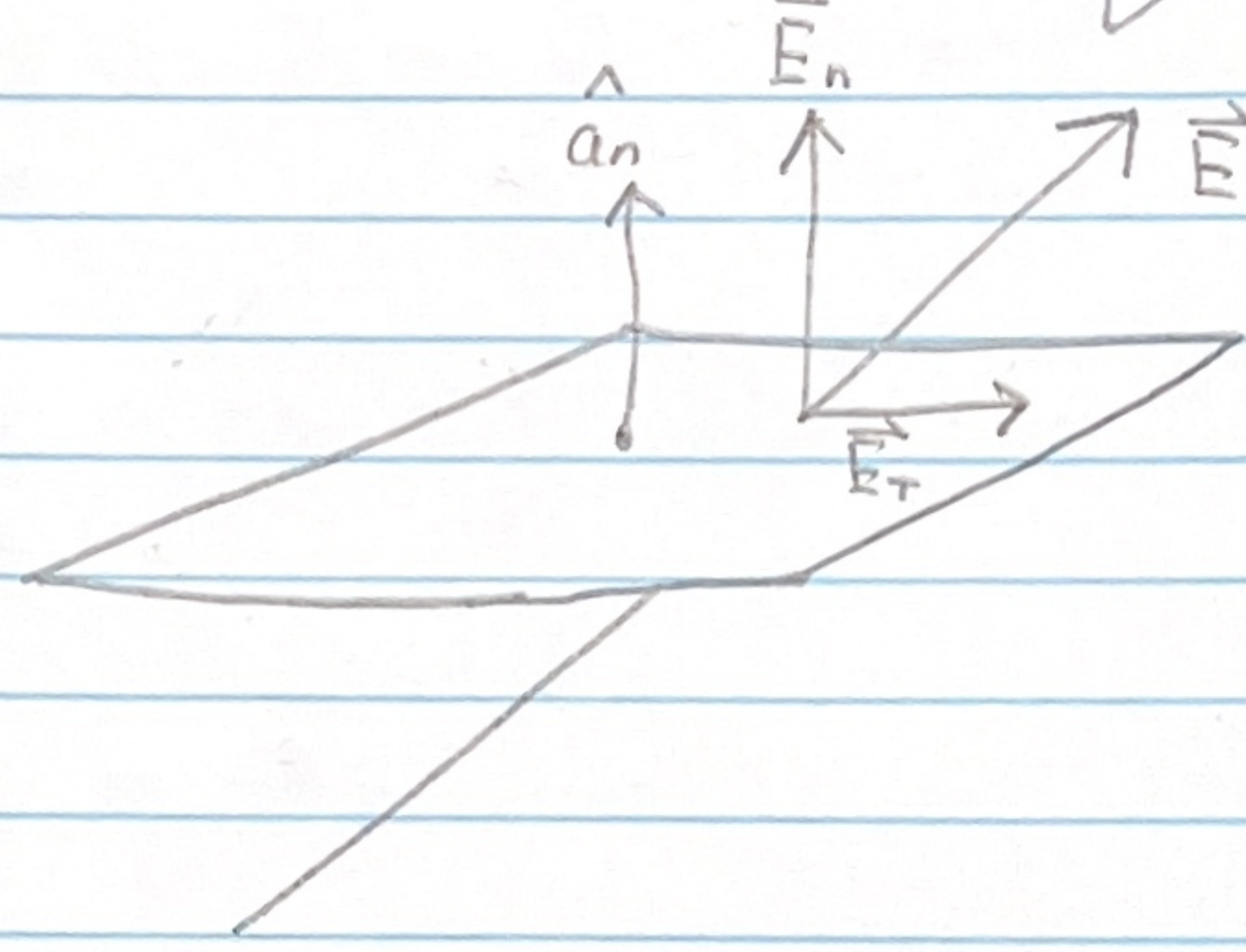
Surfaces (In Vector Calculus)

- Surface definition
- Surface Normal





$\vec{E}_n = \vec{E} \cdot \hat{a}_n$

$\vec{E} \rightarrow$ Electric field Intensity V/m

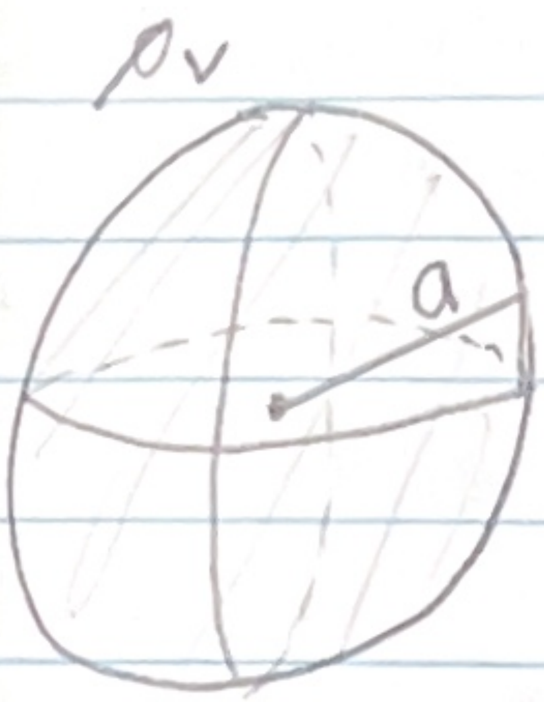
$\vec{D} \rightarrow$ Electric flux density C/m^2

$\vec{D} = \epsilon \vec{E}$

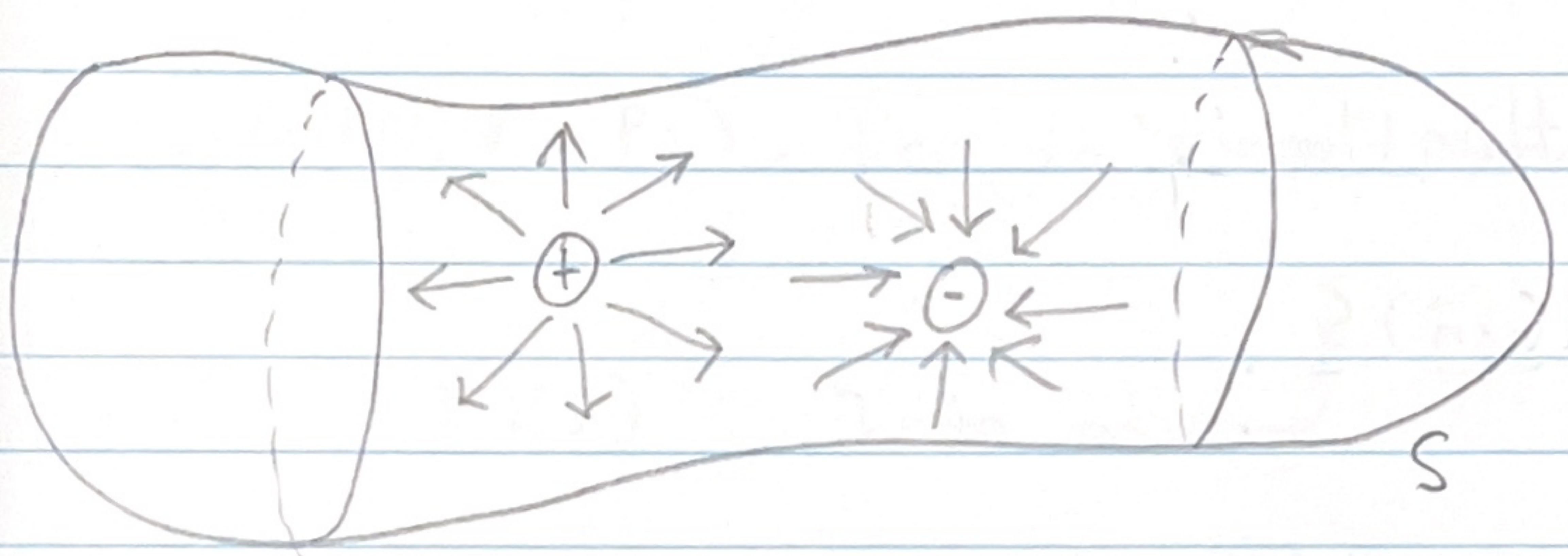
$\rightarrow \epsilon = \epsilon_r \epsilon_0$ where:

ϵ_0 = permittivity free space
ϵ_r = relative permittivity

$\vec{E} = \frac{Q}{4\pi\epsilon r^2} \rightarrow \vec{D} = \frac{Q}{4\pi r^2}$



Find D(r)

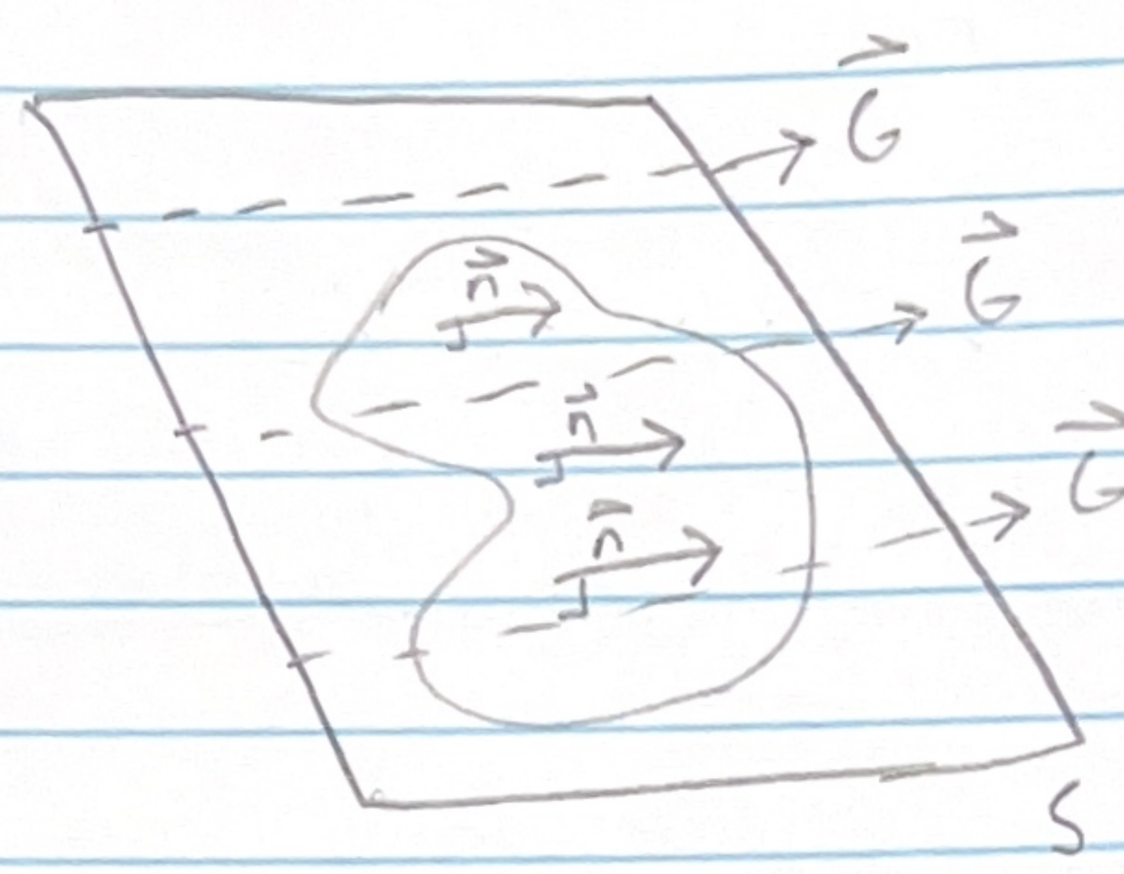


$\boxed{\text{Guass}}$ $\oint\!\!\oint_S \vec{D} \cdot \vec{ds} = Q_{encl}$

To Calculate Flux:

1. Surface
2. Flow field
3. Sign/Direction

Example

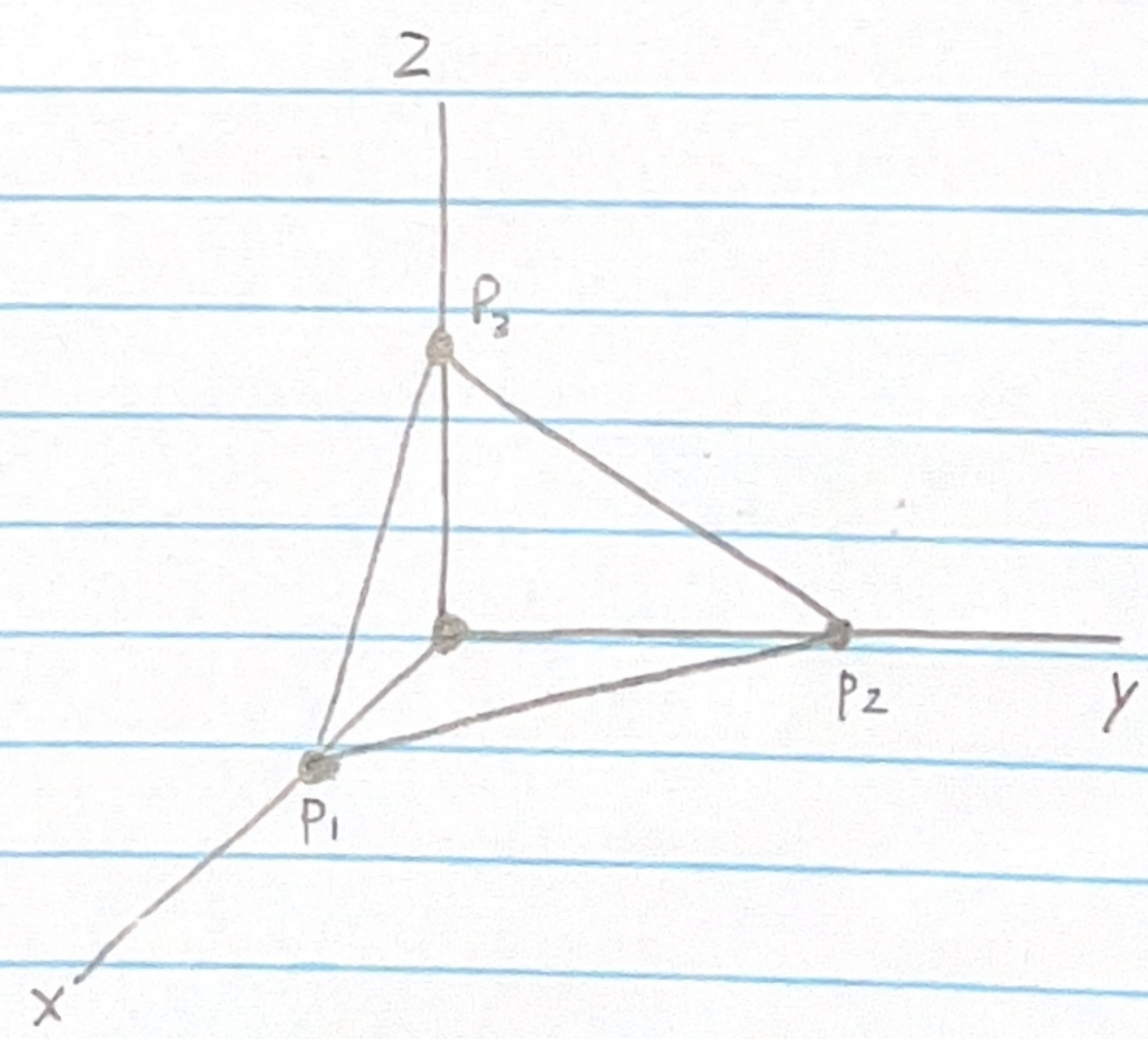


Then the Flux is:

$$\Psi = (G \cdot \vec{n}) \underbrace{S}_{\text{area of } S}$$

Given $\vec{G} = \langle A, B, C \rangle$ Find the fluxes through the faces of a tetrahedron with corners $(0,0,0)$, $(2,0,0)$, $(0,6,0)$, $(0,0,4)$ using outward normals

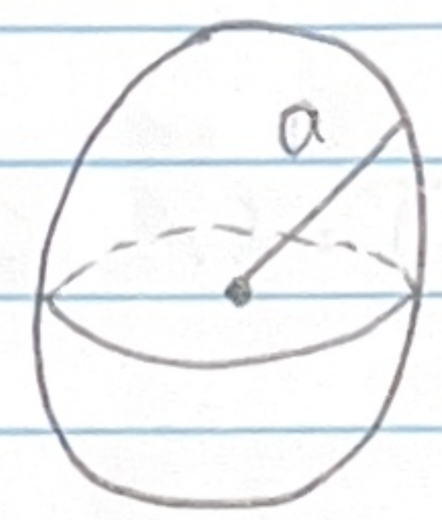
Sol'n:

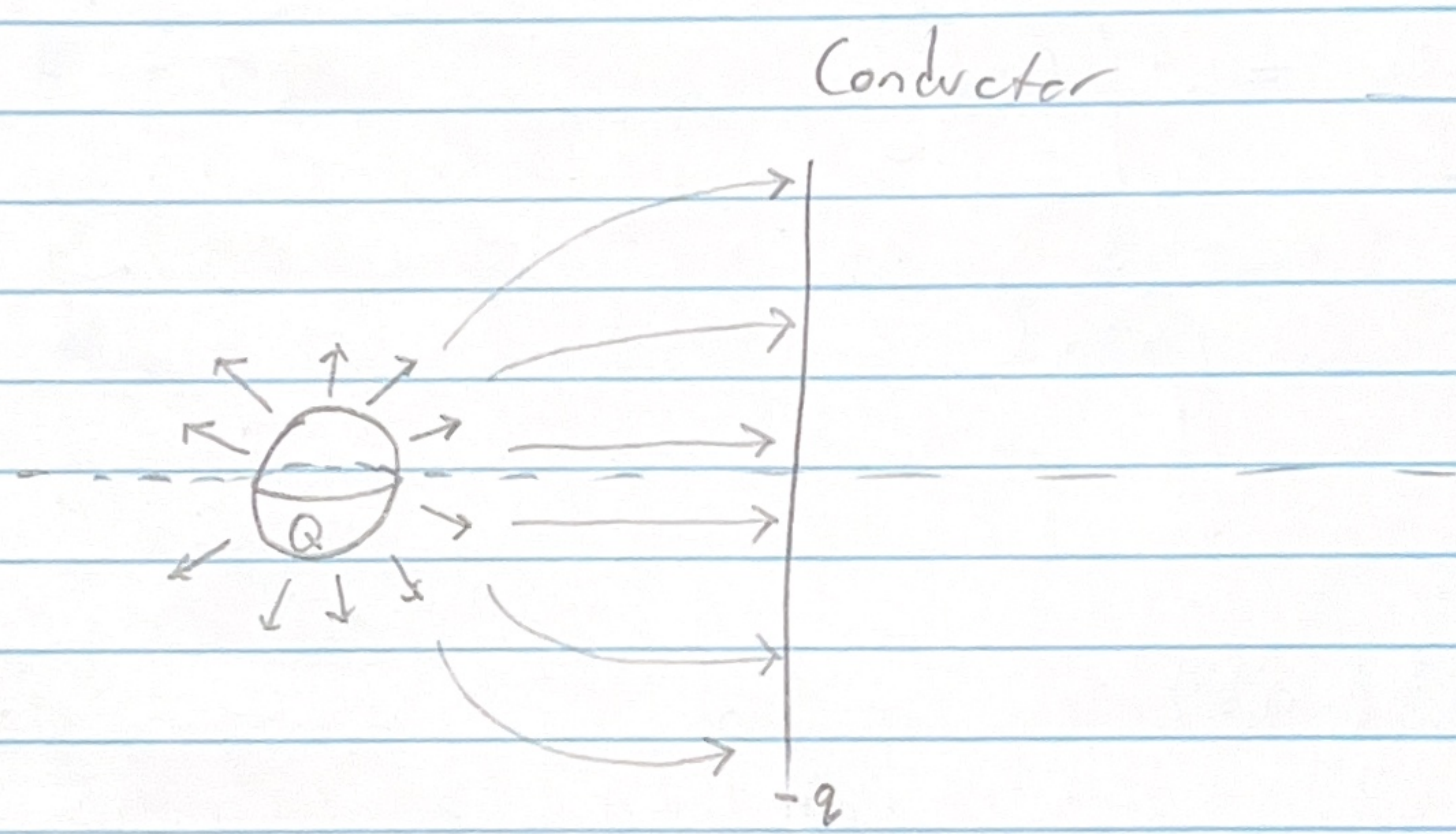


Boundary Conditions

Conducting Sphere (static) (shell)

$r < a \to \vec{E} = 0$





Ampere's Law

2/28/19
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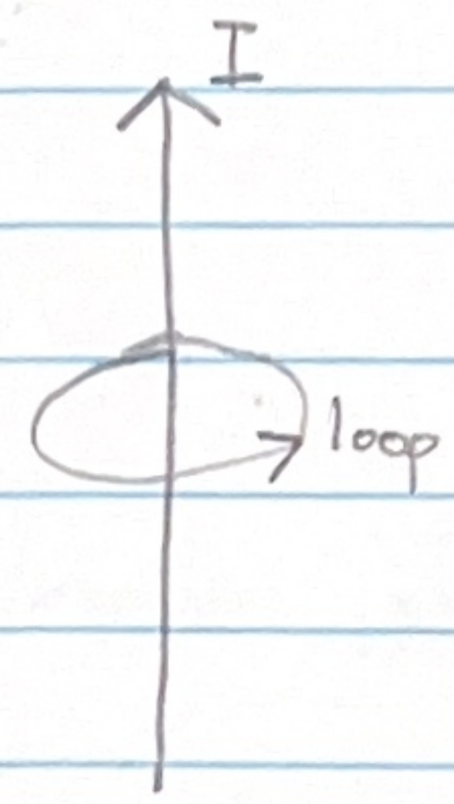
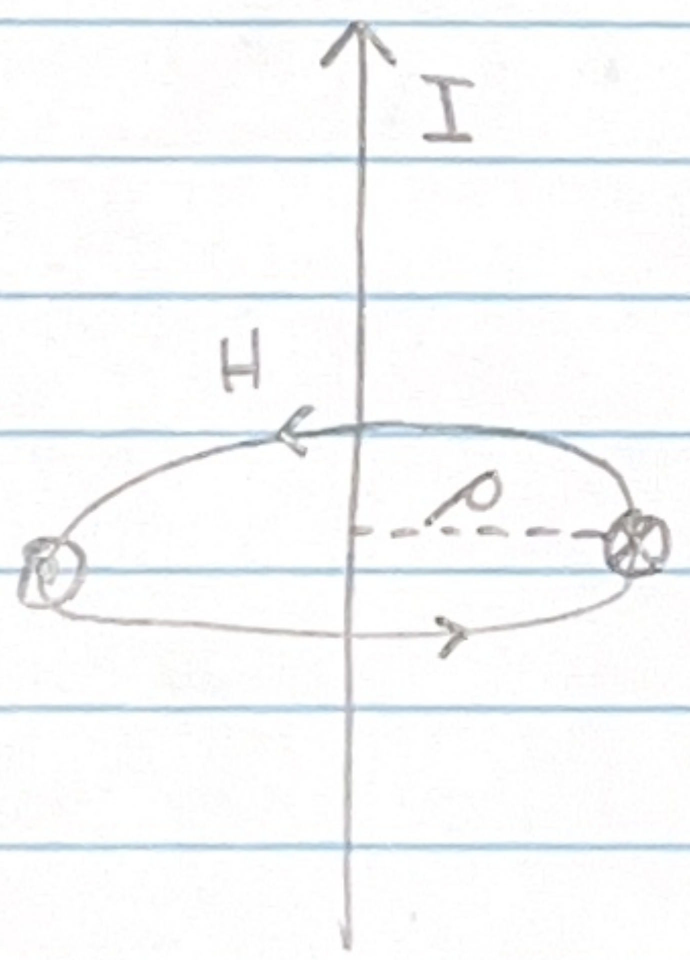
Ampere's Circuital Law

1. Closed path
2. Not time varying I or H

$$\boxed{\oint \vec{H} \cdot d\vec{L} = I}$$



Example: Infinite Current Filament



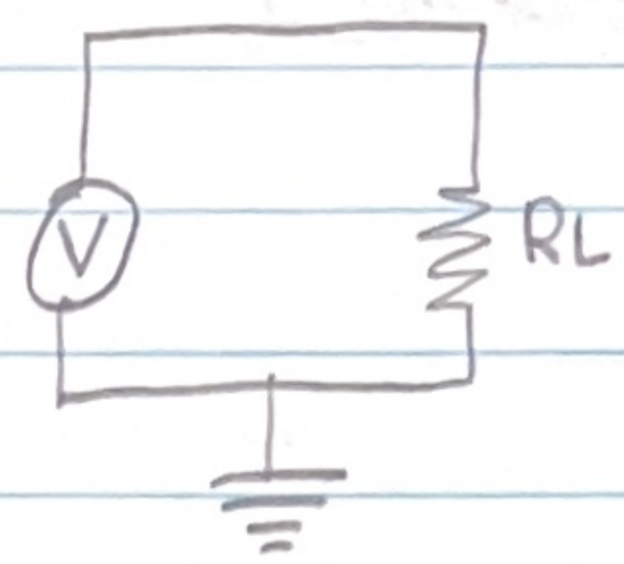
$$\oint \vec{H} \cdot d\vec{L} = I_{encl}$$

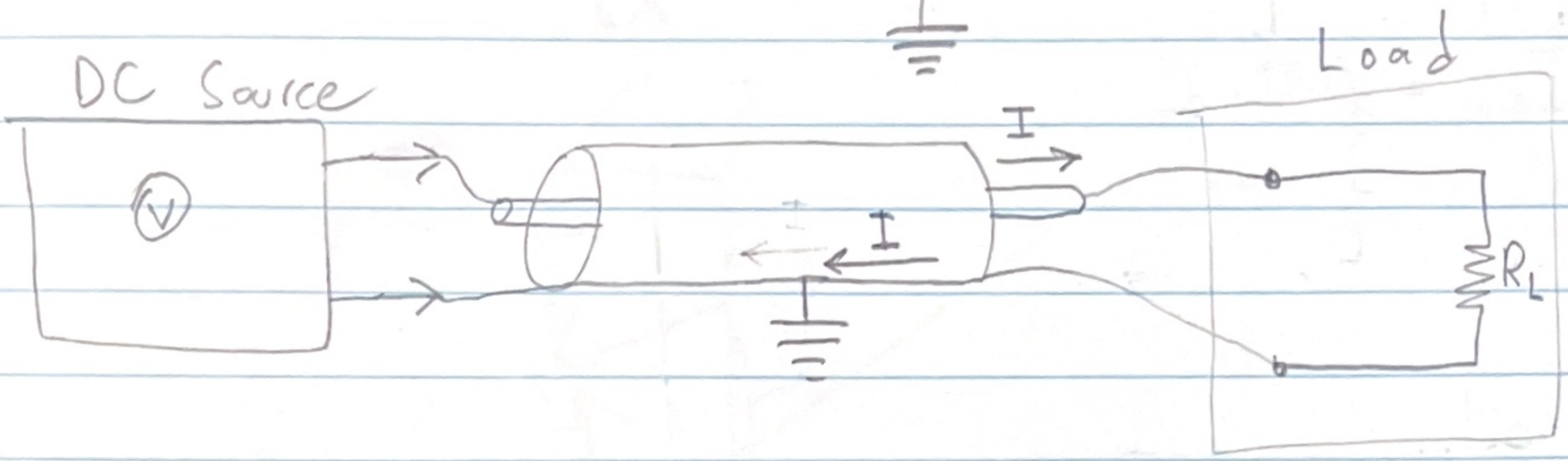
$$\int_0^{2\pi} H\hat{a}_\phi \cdot (\rho \, d\phi)\hat{a}_\phi = I$$

$$H_\phi \rho (2\pi) = I$$

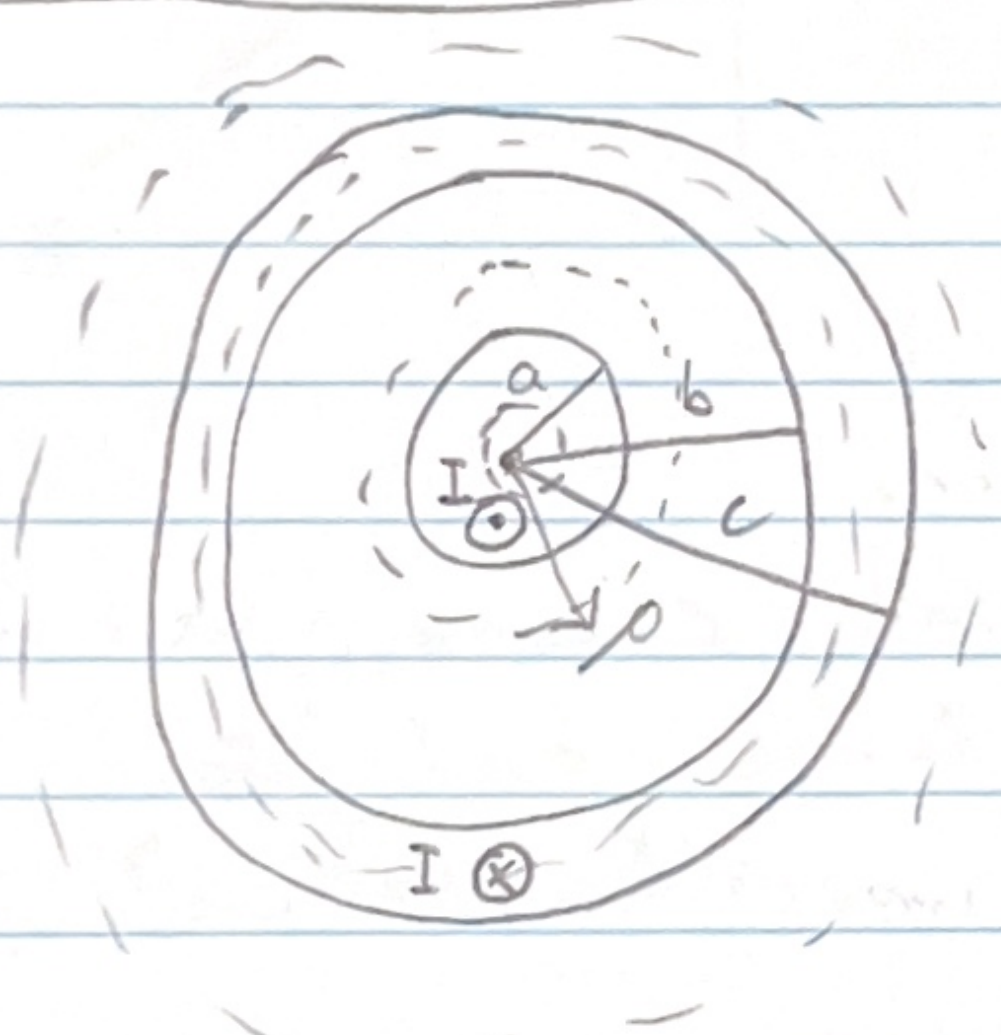
$$\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\phi$$

Coaxial Cable





Cross Section



4 Regions

Practice

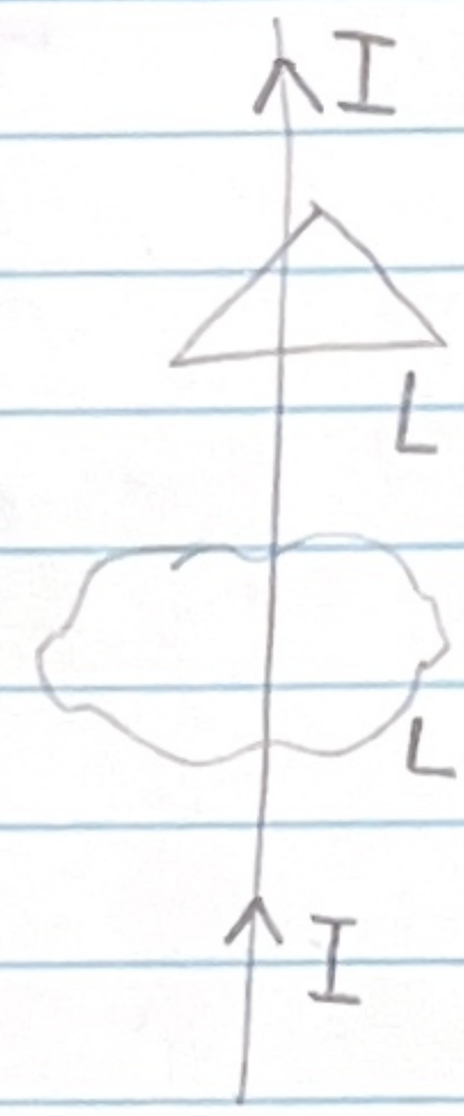
$I = \pi\rho^2.$

$\cancel{(\pi a^2)} I - \dfrac{(\rho^2 - b^2)}{(c^2 - b^2)} I$

Stokes Thereom

3/5/19
LEC

$$I_{enc} = \oint_L \vec{H} \cdot \vec{dL}$$



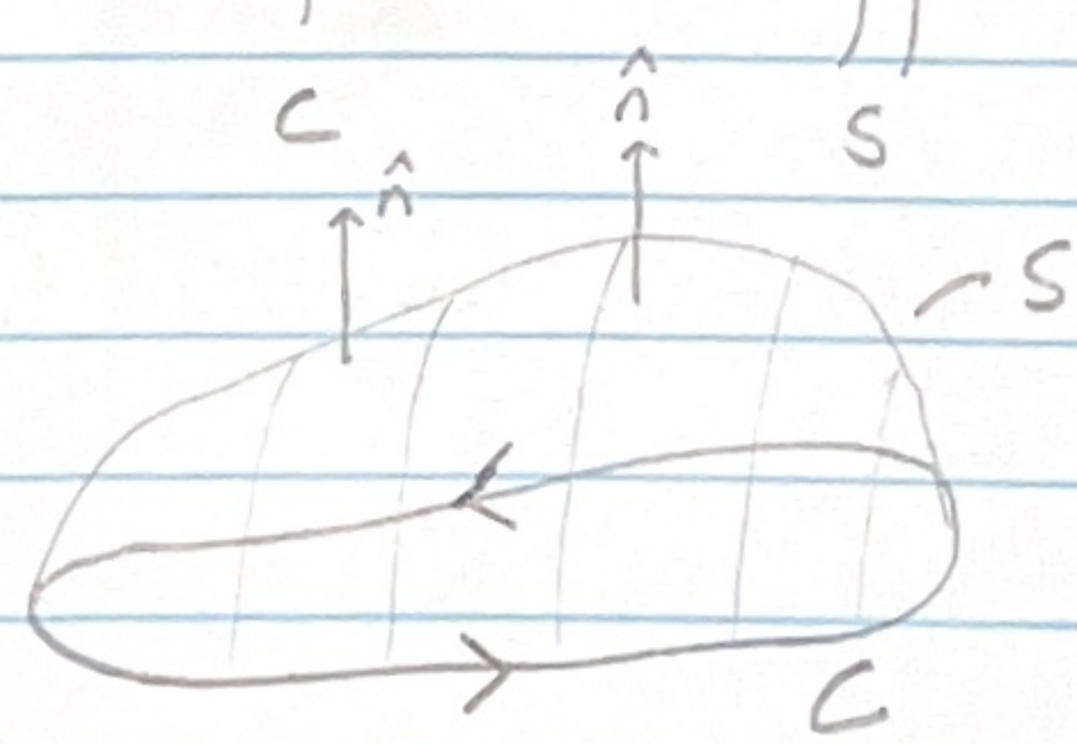
Also

$$I_{enc} = \iint_S J \cdot \hat{n}\, dS$$

Stokes Thereom

"Given any loop C and capping surface S with RHR capable orientations and for $\vec{G}$ non st at all pts of S, G we have

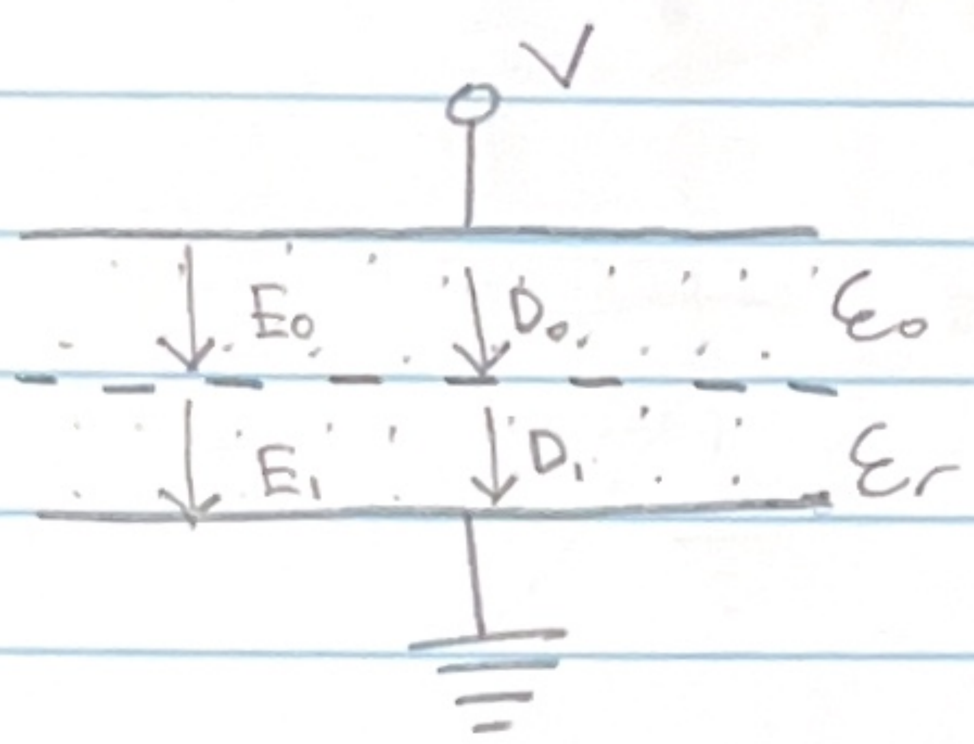
$$\oint_C \vec{G} \cdot \vec{dL} = \iint_S (\nabla \times G) \cdot \hat{n}\, dS$$



Electrostatics Review

① Dielectric and Polarization

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ "or" $\vec{D} = \epsilon_r \epsilon_0 \vec{E}$

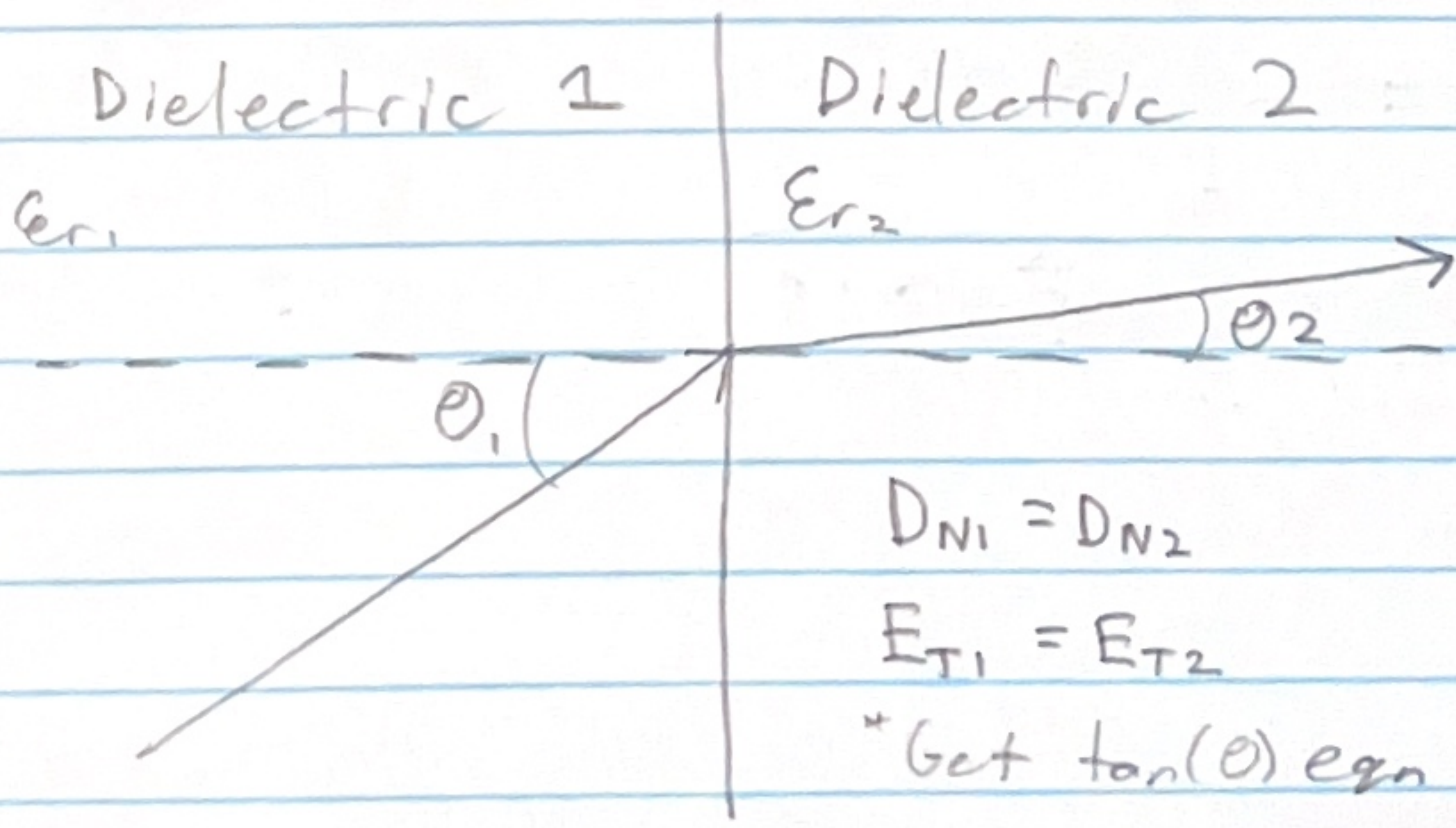


- Polarizable material changes $\vec{E}$
- Polarization opposes E
- ϵ_r takes care of this in general cases

② Boundary Conditions

Perfect Conductor	Free Space
$D = E = 0$	$D_N = \rho_s$
$V = \text{Const}$	$D_N = \epsilon_0 E_N$
	$D_T = E_T = 0$

Perfect conductor	dielectric
$D = E = 0$	$D_N = \rho_s$
$V = \text{Const}$	$D_N = \epsilon_0 \epsilon_r E_N$
	$D_T = E_T = 0$



$D_{N1} = D_{N2}$

$E_{T1} = E_{T2}$

*Get $\tan(\theta)$ eqn

$D_{T1} = \epsilon_{r1} \epsilon_0 E_{T1}$ $\quad$ $D_{T2} = \epsilon_{r2} \epsilon_0 E_{T1}$

$$\frac{D_{T1}}{\epsilon_{r1}\epsilon_0} = \frac{D_{T2}}{\epsilon_{r2}\epsilon_0}$$

Magneostatics Review

Basics

① Biot - Savart Law → $H = \oint \frac{I d\vec{L} \times \hat{a}_R}{4\pi R^2}$

Amperes Circuital Law → $\oint H \cdot dL = I_{enclosed}$

② Flux → $\Phi = \iint_S B \cdot ds = \iint_S B \cdot \hat{n} \, |ds|$ $\quad \left[\oiint \vec{B} \cdot \vec{ds} = 0 \right.$

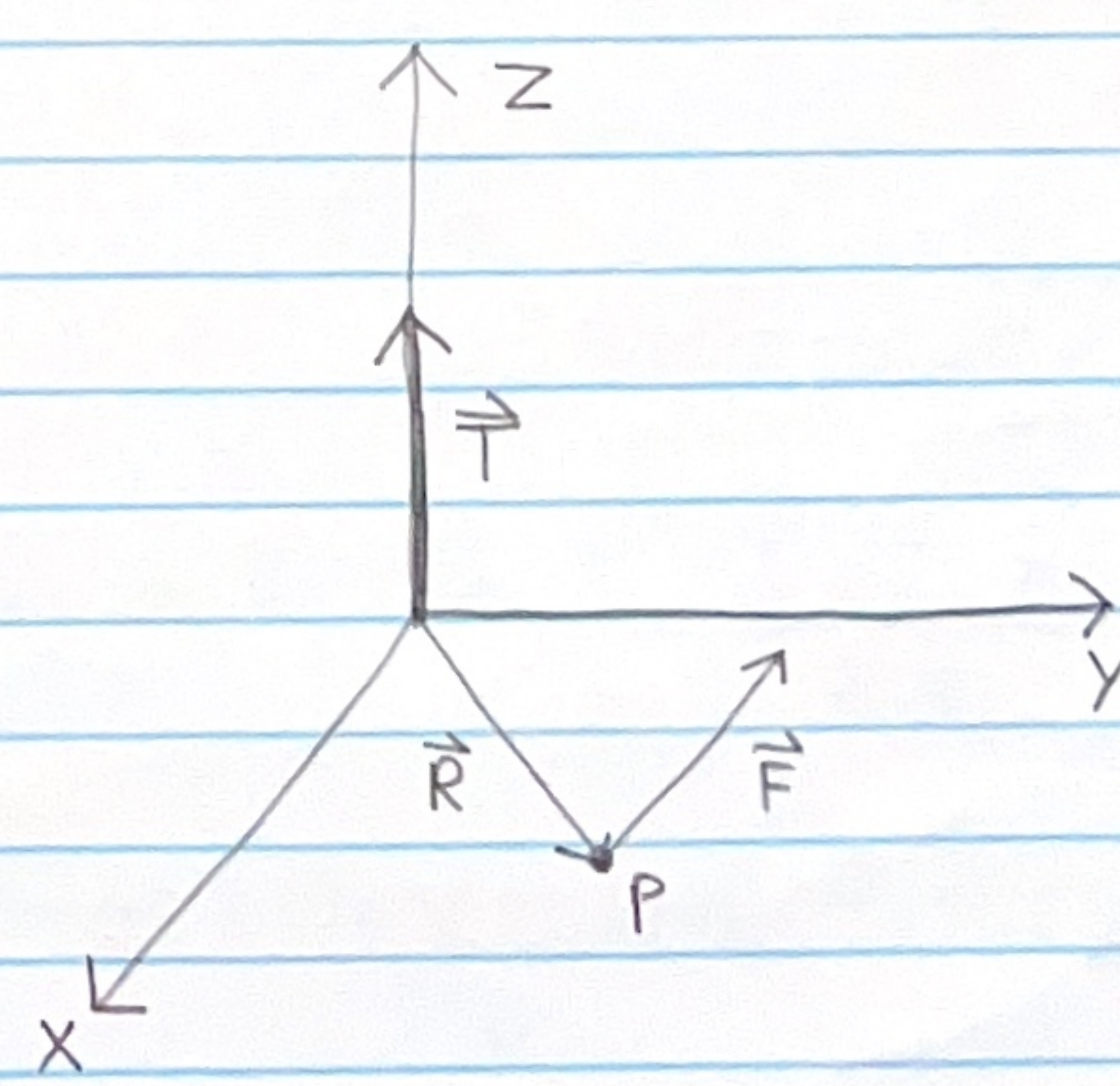
③ Force → $F = Q\vec{v} \times \vec{B}$ $\quad F = \oint I d\vec{L} \times \vec{B}$

$F = I\vec{L} \times \vec{B}$ (straight conductor)

*Assume $\mu_r = 1$ for conductors (copper)

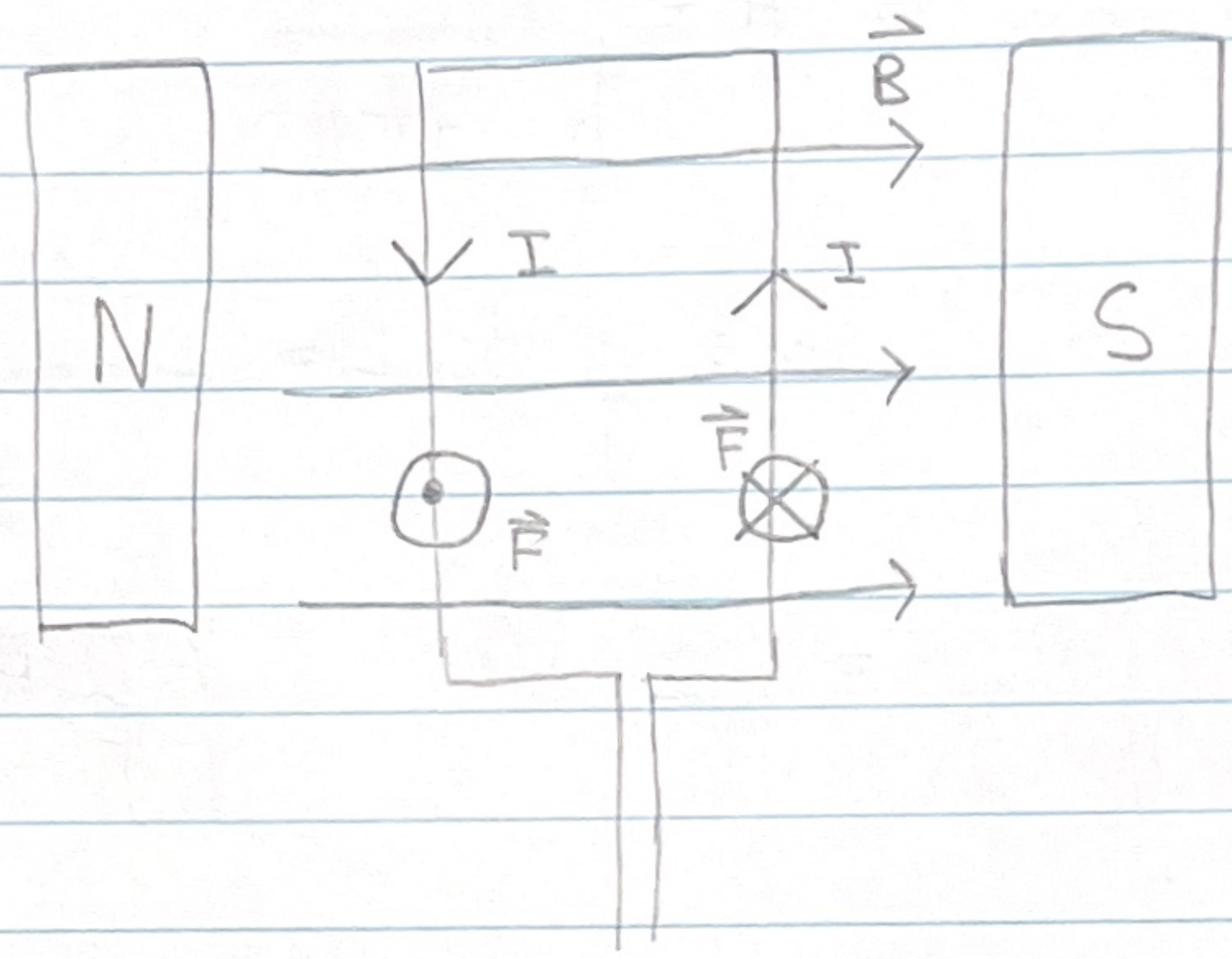
$|F| = IBL \sin\theta$

④ Torque → $T = R \times F$



*Important

"Distinguish between Net/Total Torque and .....



$F_1 + F_2 = 0$ (Net)

but $\tau \neq 0$ (Torque)

Example "infinite current (50A) on z-axis in positive $\hat{k}$ direction. Current segment from (2,0,5) to (5,0,5) and carries 6A in the $\hat{i}$ direction"

"Find torque on segment using origins

@ i) (0,0,5)

ii) (0,0,0)

iii) (3,0,0)

SOLUTION